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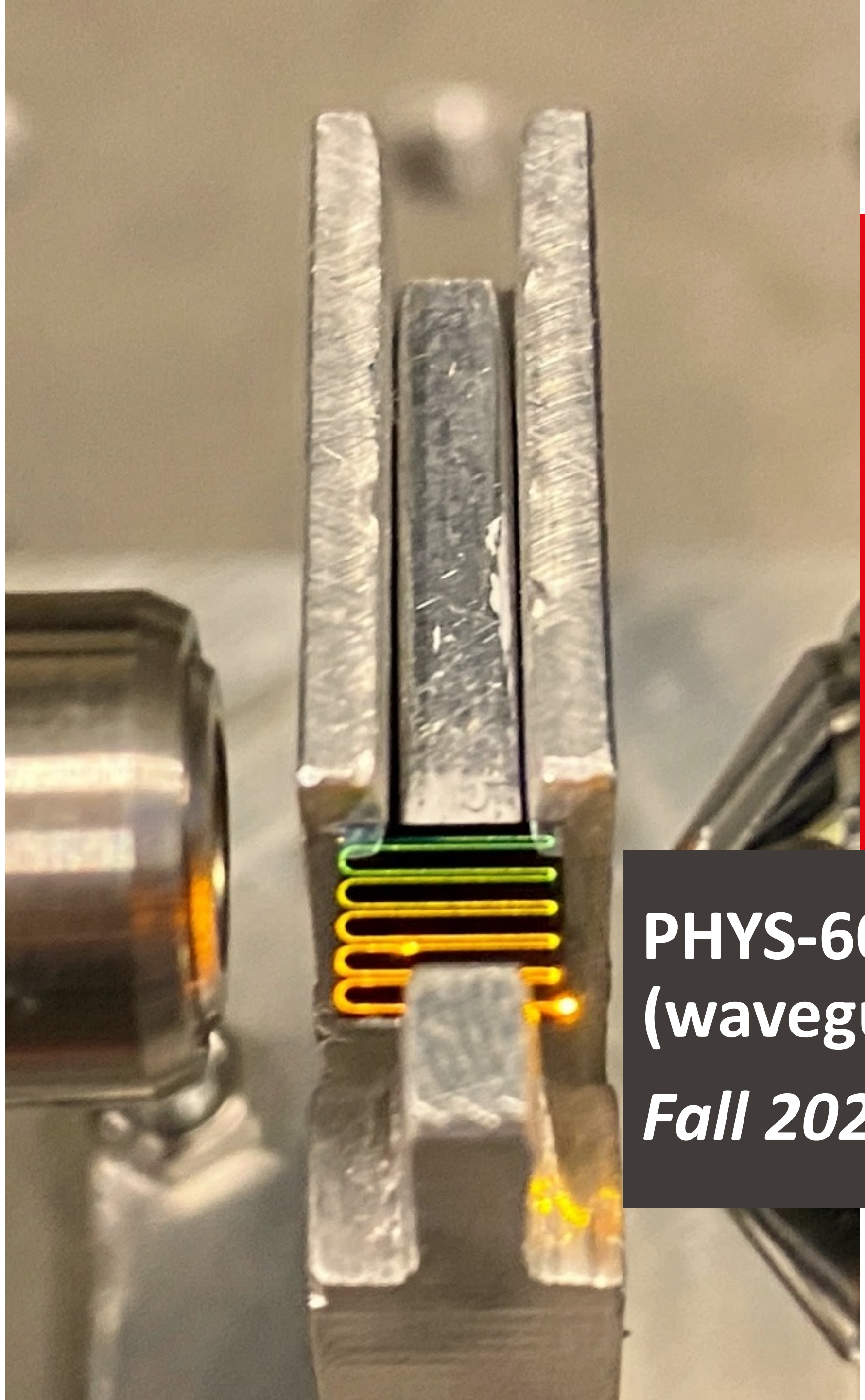

Optical Kerr effect

PHYS-607 : Nonlinear fibre (waveguide) optics

Fall 2022

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Ecole Polytechnique Fédérale de Lausanne

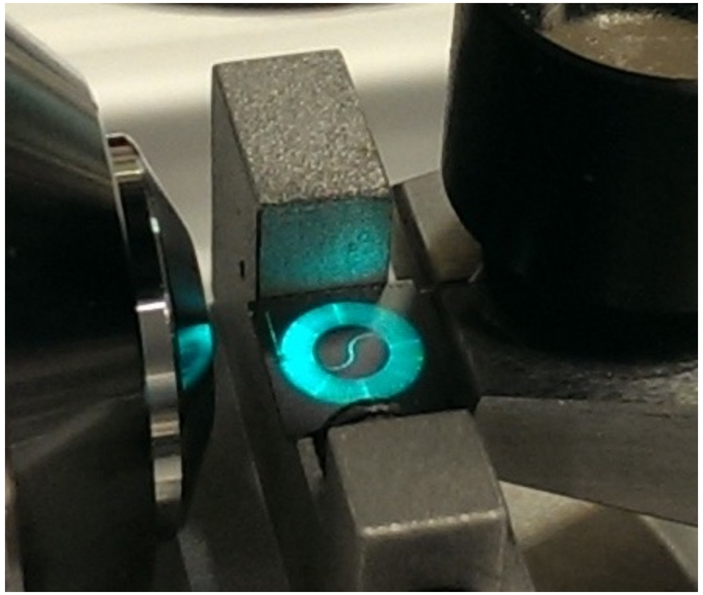
Nonlinear optics

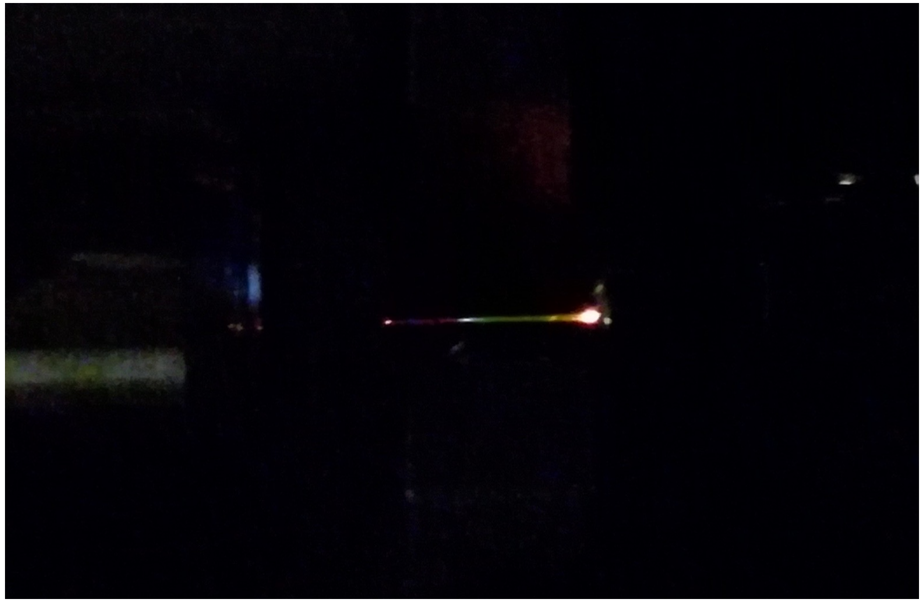
The electric response of any material becomes nonlinear with sufficiently large electromagnetic fields

A signature of nonlinearity is the generation of new frequencies

It enables a light stream to influence other light streams

Opens many possibilities for optical processing





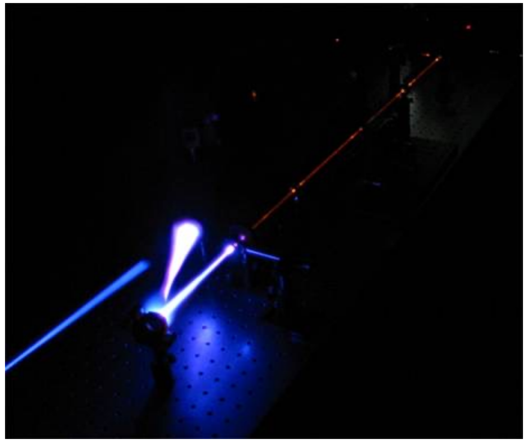
A quick time line

1875: John Kerr reports the first nonlinear effect: double refraction in solid and liquid dielectrics in a strong electrostatic field.

Kerr, John (1875). "A new relation between electricity and light: Dielectrified media birefringent". Philosophical Magazine. 4. ***50*** *(332)*

1960: T.H. Maiman demonstrates the first working laser

1961: Franken et al. make the first observation of wavelength conversion by second harmonic generation



1962: Terhune et al observe third harmonic generation

1973: Hasegawa et al. predict the generation of solitons in fibers

1980: Mollenauer et al. experimentally demonstrate solitons in fibers

… many more, nonlinear optics is present in many systems

Applications

Laser and optical sources (green laser pointer, Modelocked, Q-switched, regenerative lasers …)

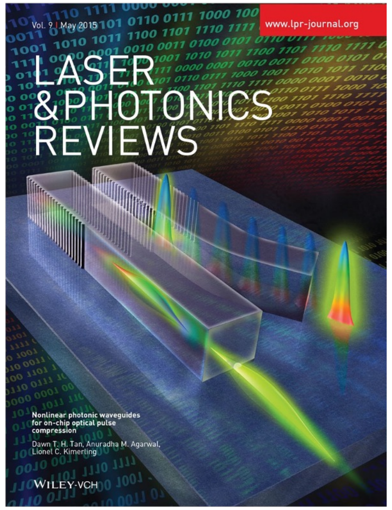
Ultra fast optics (Pulse compression, pulse characterization….)

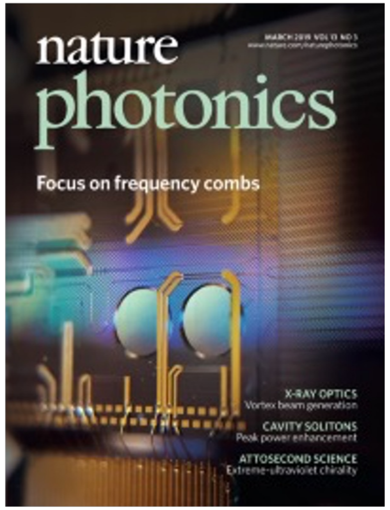
Wavelength conversion (supercontinuum, wave mixing …)

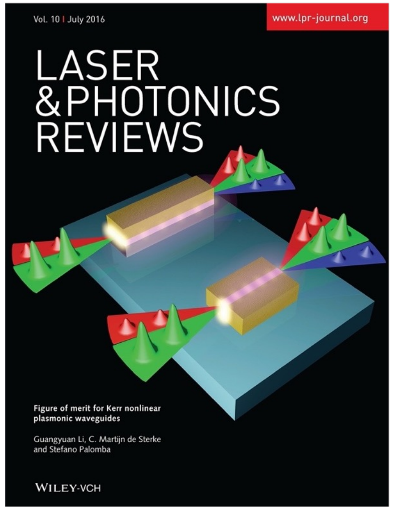
Optical communication (amplification, regeneration, dispersion compensation …)

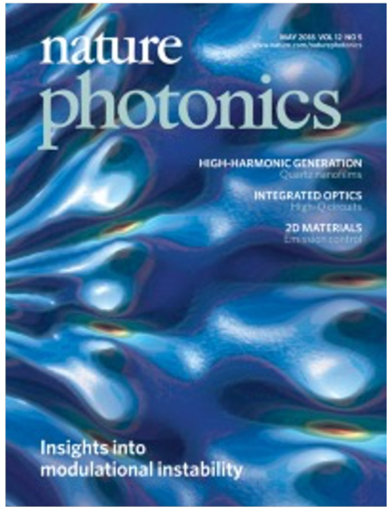
Imaging and health sciences (optical coherence tomography, nonlinear fluorescence …)

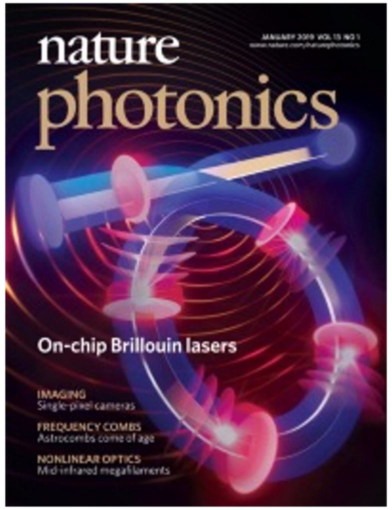
Remote applications (LiDAR …)



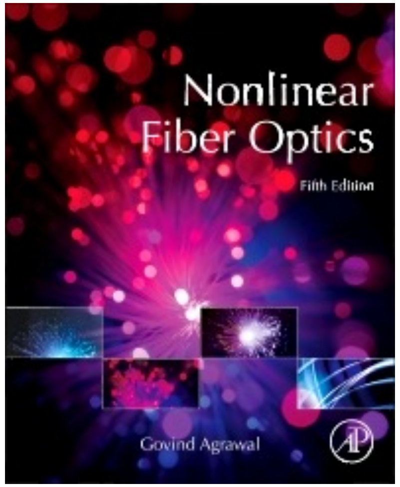




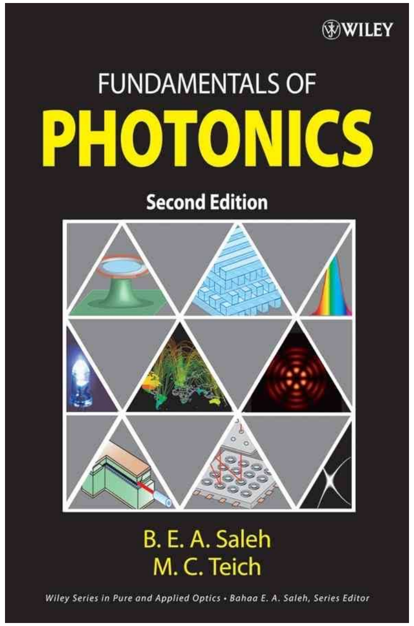


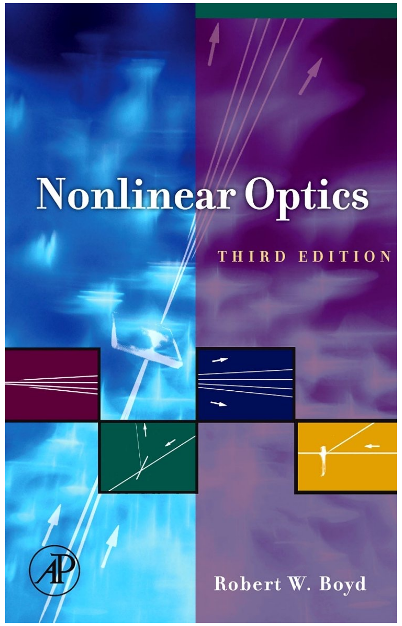


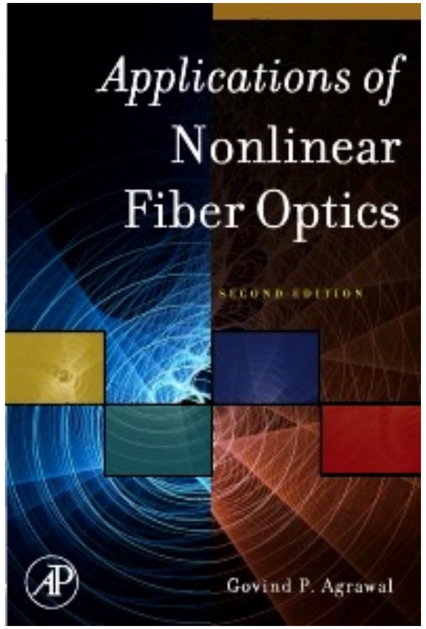
Textbook



G. P. Agrawal, **Nonlinear fiber optics**, 5th edition, Academic Press, 2013







Sign convention

Our electric field definition is the one used in Agrawal's book, with a negative carrier frequency.

$$\mathbf{E}(z,\ t) = \frac{1}{2} A_0(t) \exp\left[j\left(\underbrace{-\omega_0 t + \beta z}_{} \right) \right] + c.c.$$

Envelope (real)

Phase

In this case the Fourier transform pair is defined as:

$$\widetilde{U}(\omega) = \int_{-\infty}^{\infty} U(T) \exp\left(j\omega T \right) dT \qquad U(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widetilde{U}(\omega) \exp\left(-j\omega T \right) d\omega$$

The instantaneous frequency as : $\omega(t) = -\dfrac{\partial \varphi(t)}{\partial t}$

Outline

Module 1 – Overview of third order nonlinear parametric processes

Module 2 – Self phase modulation

Module 3 – Nonlinear pulse propagation

Module 4 – Solitons

Module 5 – Modulation instability

Module 6 – Solving the NLSE

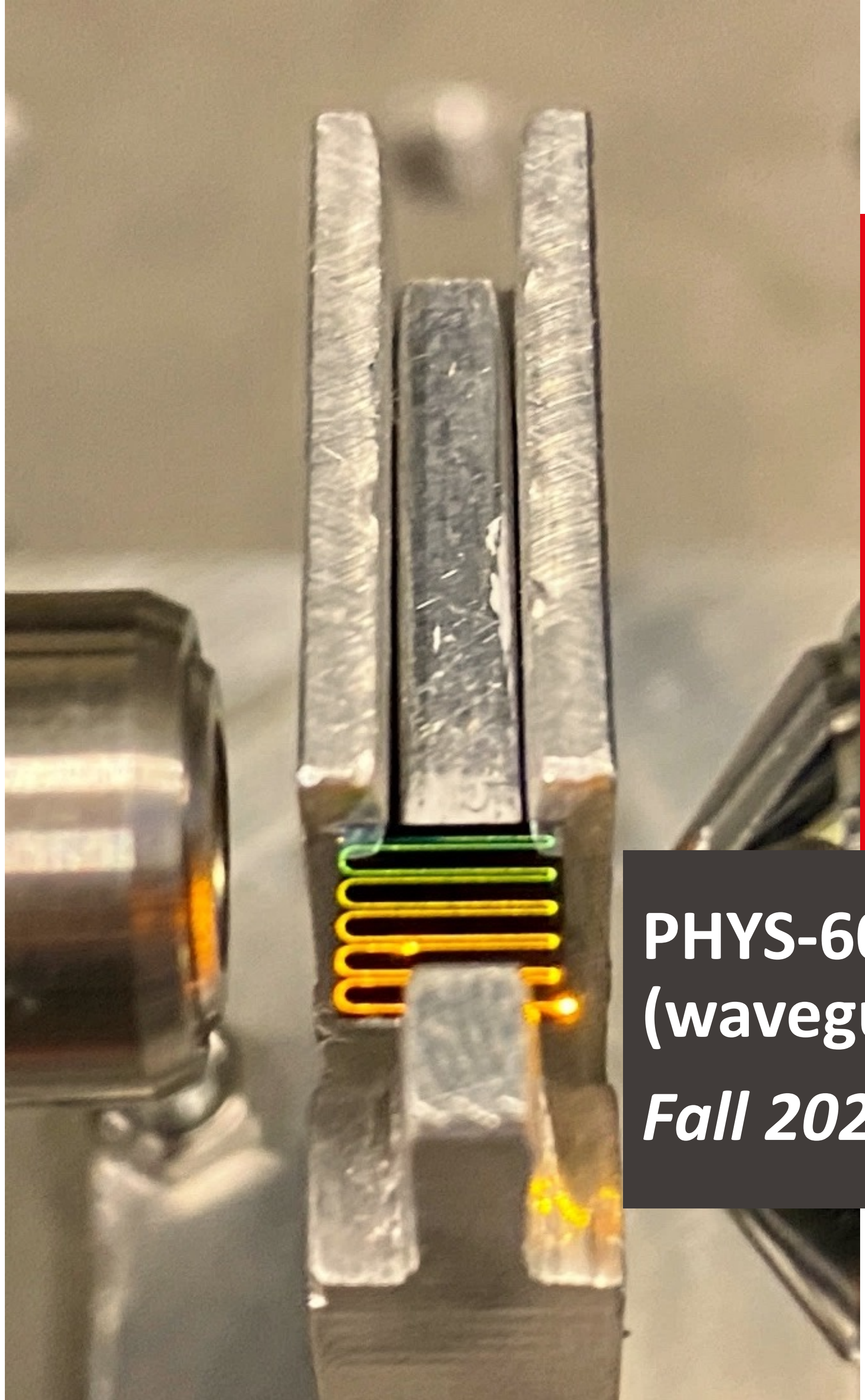
Module 7 – Multiwavelength effects: cross phase modulation and four-wave mixing

Module 8 – Parametric amplification

Module 9 – Supercontinuum generation

(Extra – Self steepening)

EPFL



Optical Kerr effect

Module 1 - Overview of 3rd order nonlinear parametric processes

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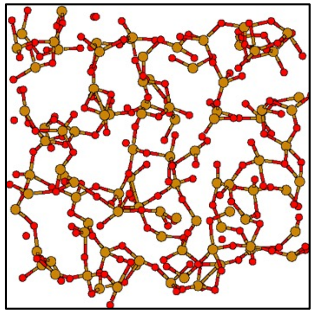
Overview of processes in a third-order nonlinear medium

Here we will examine the optical properties of a nonlinear medium in which the 3rd order nonlinearity dominates.

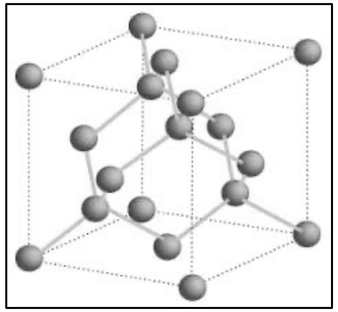
- This is the case for materials which are centrosymmetric on the macroscopic scale.

SiO_2, ZBLAN, ChG, SiN, aSi …



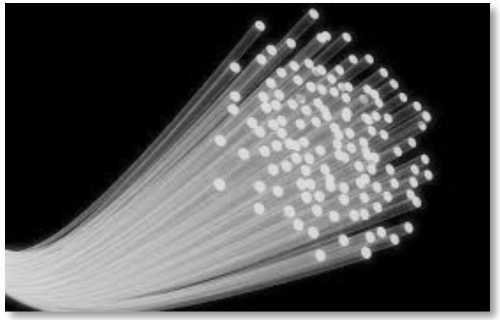
$< \chi^{(2)} > = 0$

Si

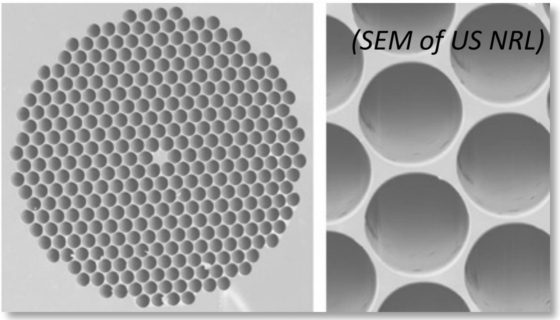


$\chi^{(2)} = 0$

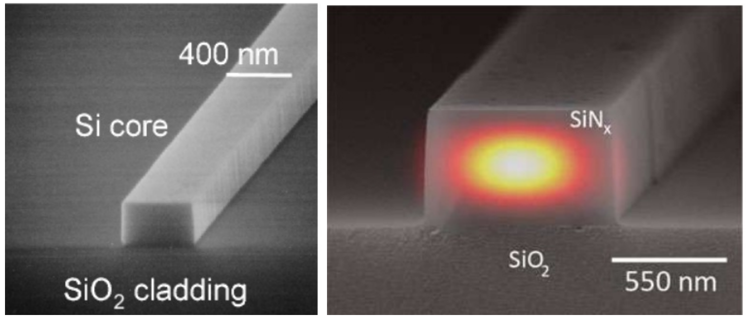
$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E} + \varepsilon_0 \chi^{(2)} \mathbf{E}^2 + \underline{\varepsilon_0 \chi^{(3)} \mathbf{E}^3}$$



Step index fiber



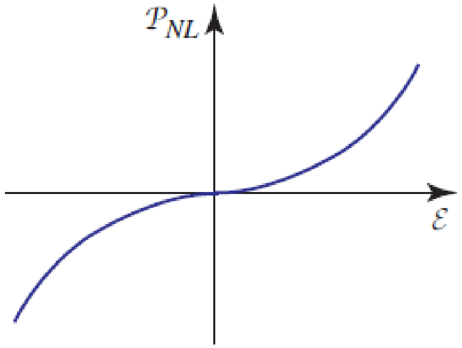
Microstructured fiber



Silicon compound waveguides

Nonlinear propagation

$$\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)} \mathbf{E}^{\mathbf{3}}$$



Consider a single harmonic field:

$$\mathbf{E}(t) = \mathrm{Re}[E(t)\exp(-j\omega t)] = \frac{1}{2}[E(t)\exp(-j\omega t) + c.c]$$

The resulting nonlinear polarization is:

$$\mathbf{P}_{NL}(t) = \frac{1}{2}\left[P_{NL,\omega}\exp(-j\omega t) + P_{NL,3\omega}\exp(-j3\omega t)\, c.c\right]$$

$$P_{NL,\omega} = \frac{3}{4}\varepsilon_0 \chi^{(3)} E^2 E^*$$ **ω frequency source**

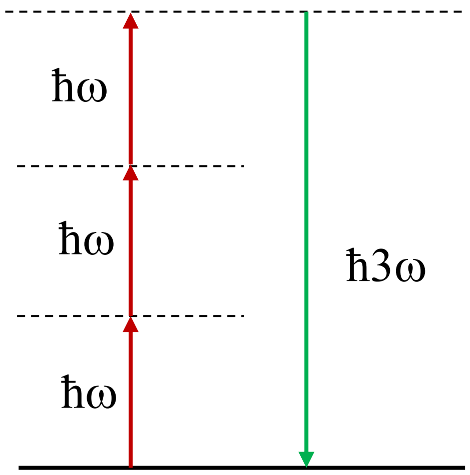
$$P_{NL,3\omega} = \frac{1}{4}\varepsilon_0 \chi^{(3)} E^3$$ **3ω frequency source**

Third harmonic generation

$$\mathbf{P}_{NL}(t) = \frac{1}{2}\left[P_{NL,\omega}\exp(-j\omega t) + P_{NL,3\omega}\exp(-j3\omega t)\; c.c\right]$$

$$P_{NL,\omega} = \frac{3}{4}\varepsilon_0\chi^{(3)}E^2E^*$$

$$\boxed{P_{NL,3\omega} = \frac{1}{4}\varepsilon_0\chi^{(3)}E^3}$$



The 3rd harmonic generation is difficult to achieve, since the source term must propagate at the same velocity as the field term $\Longrightarrow$ $3\beta_\omega = \beta_{3\omega}$

Pertubation on the ω wave

$$P_{NL,\omega} = \frac{3}{4}\varepsilon_0\chi^{(3)}E^2E^*$$

$$P_{NL,\omega} = \frac{3}{4}\varepsilon_0\chi^{(3)}|E|^2E$$

$$\propto I = \frac{n\varepsilon_0 c_0}{2}|E|^2$$

This wave induces a change in the real and the imaginary part of the susceptibility χ:

$$\mathbf{P}(t) = \varepsilon_0\chi^{(1)}\mathbf{E}(t) + \mathbf{P}_{NL}(t) \approx \varepsilon_0\chi^{(1)}\mathbf{E}(t) + \varepsilon_0\Delta\chi\mathbf{E}(t)$$

Nonlinear polarization as a perturbation

with $\Delta\chi = \frac{3}{4}\chi^{(3)}|E|^2$

Pertubation on the ω wave

We know that the complex refractive index is given by $\varepsilon = \underline{\tilde{n}}^2 = 1 + \chi = \left(\tilde{n} + j\frac{\tilde{\alpha}}{2k_0}\right)^2$

This is similar to linear case, except that in this case both refractive index and loss are perturbed:

$$\tilde{n} = n + \Delta n \qquad\qquad \tilde{\alpha} = \alpha + \Delta\alpha$$

The change in $\Delta\chi$ thus leads to a change:

$$2n\left(\Delta n + j\frac{\Delta\alpha}{2k_0}\right) = \Delta\chi$$

$$2n\Delta n + jn\frac{\Delta\alpha}{k_0} = \frac{3}{4}\chi^{(3)}|E|^2$$

Pertubation on the ω wave

By identification from $2n\Delta n + jn\dfrac{\Delta\alpha}{k_0} = \dfrac{3}{4}\chi^{(3)}|E|^2$ we get:

$$\Delta n = \frac{3}{8n}\mathrm{Re}\left(\chi^{(3)}\right)|E|^2 \equiv n_2 I \quad \text{with} \quad n_2 = \frac{3}{4n^2\varepsilon_0 c_0}\mathrm{Re}\left(\chi^{(3)}\right)$$

$$\Delta\alpha = \frac{3k_0}{4n}\mathrm{Im}\left(\chi^{(3)}\right)|E|^2 \equiv \beta I \quad \text{with} \quad \beta = \frac{3k_0}{2n^2\varepsilon_0 c_0}\mathrm{Im}\left(\chi^{(3)}\right)$$

$$\left(\text{Recall: } I = \frac{n\varepsilon_0 c_0}{2}|E|^2\right)$$

Kerr effect and nonlinear absorption

$$n(I) = n_0 + n_2 I$$

n_2 **is the nonlinear refractive index**

$$\alpha(I) = \alpha_0 + \beta I$$

β **is the nonlinear absorption coefficient**

The intensity dependent (real) refractive index is the **Kerr effect**

- n_2 = 2.7 10^{-20} m^2/W in silica (SiO_2)
- n_2 = 6 10^{-18} m^2/W in silicon (Si)
- n_2 = 2.4 10^{-19} m^2/W in silicon nitride (Si_3N_4)

The intensity-dependent absorption is also called **two-photon absorption**

- It is relatively small for SiO_2, Si_3N_4 and is often ignored
- β = 0.5 cm/GW for Si at 1550 nm

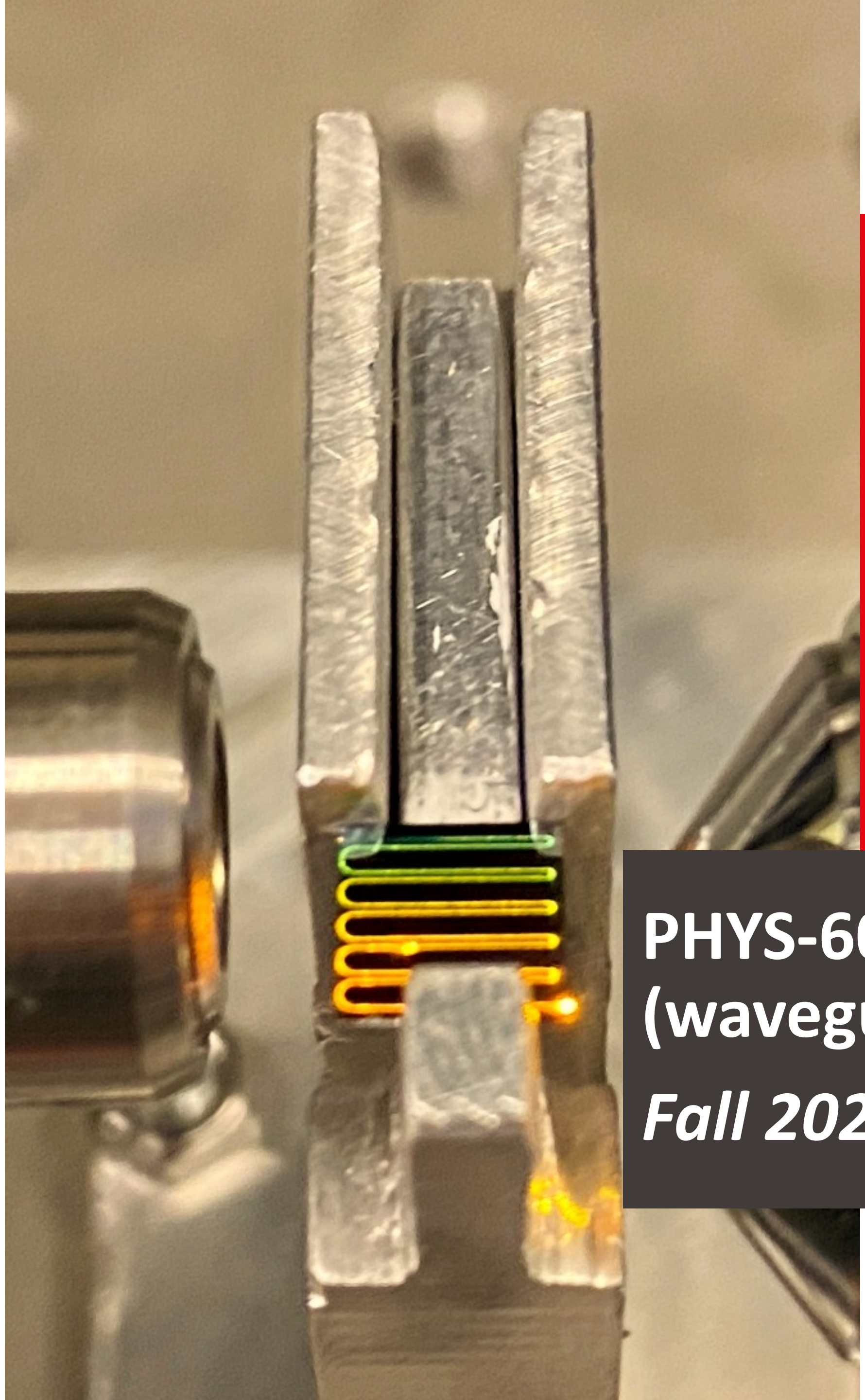
Origin of the nonlinear refractive index n_2

Mechanism	n_2 (m²/W)	$\chi^{(3)}_{1111}$ (m²/V²)	Response time (s)
Electronic polarization	10^{-20}	10^{-22}	10^{-15}
Molecular orientation	10^{-18}	10^{-20}	10^{-12}
Electrostriction	10^{-18}	10^{-20}	10^{-9}
Thermal effects	10^{-10}	10^{-12}	10^{-3}

There are various origins to the nonlinear refractive index. Those that yield the largest n_2 typically have a slow response time.

In amorphous dielectric materials (silica, soft glasses, etc…) it has essentially 2 origins:

- The non-resonant electronic contribution: weak but is always present and has no observed frequency limit (fast response).
- The refractive index change due to the electrostriction induced by a gradient of intensity: dominant contribution but is observed only in focused beams and has a frequency limit of approx. 300 MHz in standard optical fibres (slow response).



Optical Kerr effect

Module 2 – Self phase modulation

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Self phase modulation (SPM)

As a result of the Kerr effect, an intensity varying propagating wave can act upon itself and experience an *intensity related phase shift.*

Reminder

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\hat{\boldsymbol{e}} A_0[j(-\omega_0 t + \beta z + \phi_0)]\right] \qquad \beta = n k_0 \qquad n(I) = n_0 + n_2 I$$

The accumulated phase of the wave is: $\varphi_a(z) = \beta z$

$$\varphi_a(z) = n k_0 z$$

$$\varphi_a(z) = \frac{2\pi}{\lambda_0}(n_0 + n_2 I)z$$

An intensity dependent phase shift builds up along a wave profile: signal is phase modulated by its own intensity – ***self phase modulation***

Self phase modulation (SPM)

$$\varphi_a(z) = \frac{2\pi}{\lambda_0}(n_0 + n_2 I)z$$

$$\varphi_a(z) = \varphi_0 + \Delta\varphi(z) \quad \text{with} \quad \Delta\varphi(z) = \frac{2\pi}{\lambda_0} n_2 I z$$

The total additional phase shift φ_{NL} caused by self phase modulation after distance L :

$$\varphi_{NL} = \int_0^L \Delta\varphi(z)dz = \int_0^L \frac{2\pi}{\lambda_0} n_2 I(z)dz$$

$$\varphi_{NL} = \frac{2\pi}{\lambda_0}\frac{n_2}{A_{eff}}\int_0^L P(z)dz$$

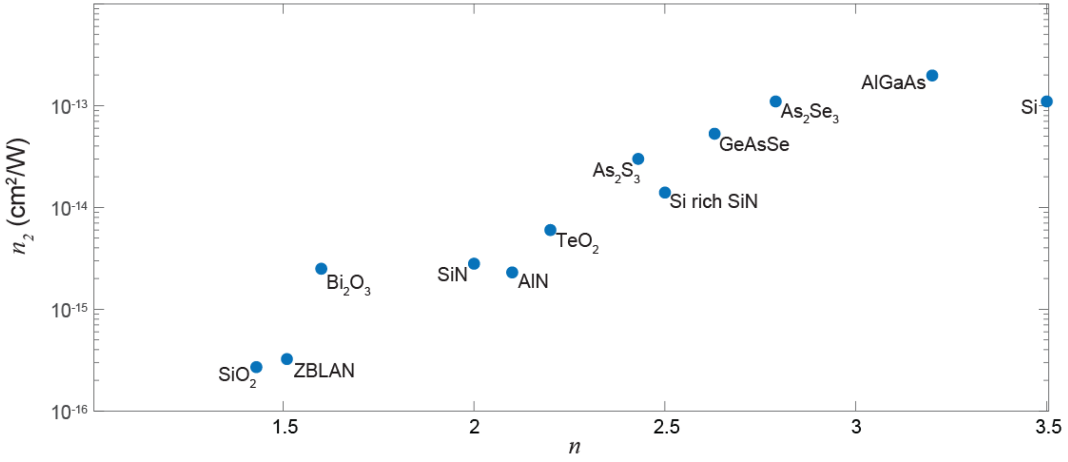
$$\varphi_{NL} = \frac{2\pi}{\lambda_0}\frac{n_2}{A_{eff}} P_0 L_{eff}$$

Nonlinear coefficient

A **nonlinear coefficient** γ is defined such that $\varphi_{NL} = \gamma P_0 L_{eff}$

Therefore $\gamma = \dfrac{2\pi}{\lambda_0} \dfrac{n_2}{A_{eff}}$ in $(\mathrm{W}^{-1}\mathrm{km}^{-1})$

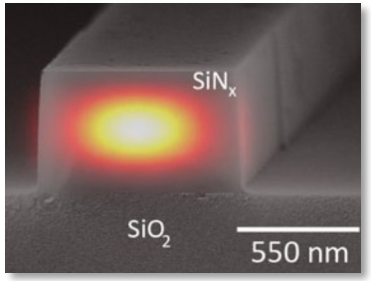
- Depends on (some) material properties: n_2
- Depends on (some) waveguiding properties: A_{eff}

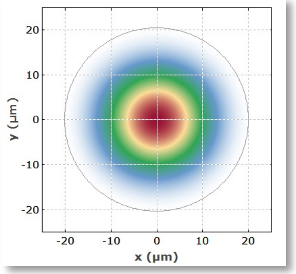


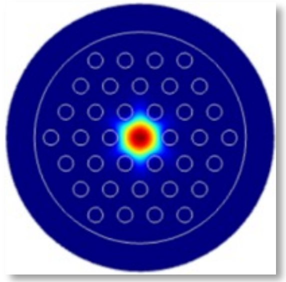
A typical value in standard silica fibres is $\gamma \sim 1\ \mathrm{W}^{-1}\ \mathrm{km}^{-1}$

A nonlinear length can be defined, such as: $L_{NL} = \dfrac{1}{\gamma P_0}$

- Length for which $\varphi_{\mathrm{NL}} = 1$







Maximum phase shift and spectral broadening

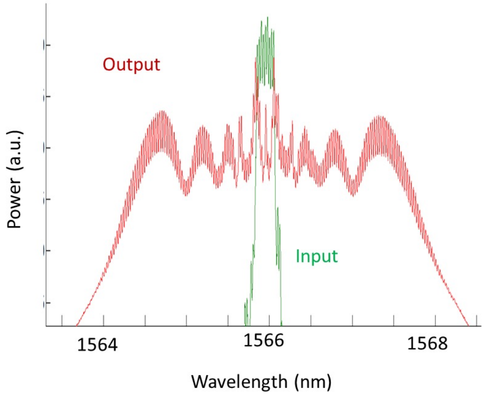
Assume the input signal has a time varying envelope: $A(0,t) = \sqrt{P_p}\, U(0,t)$

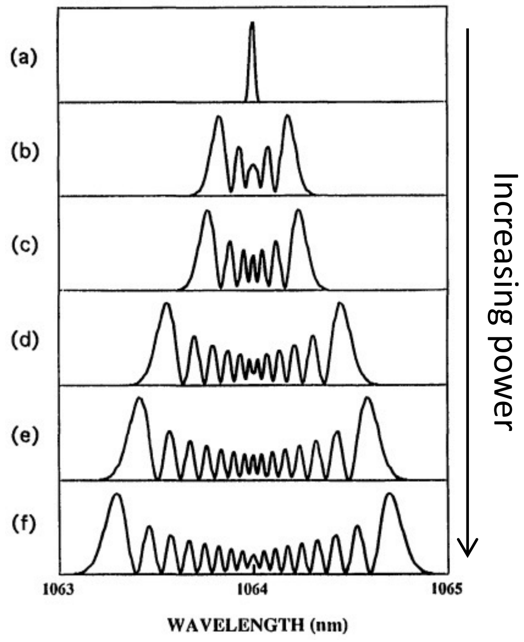
Normalized envelope

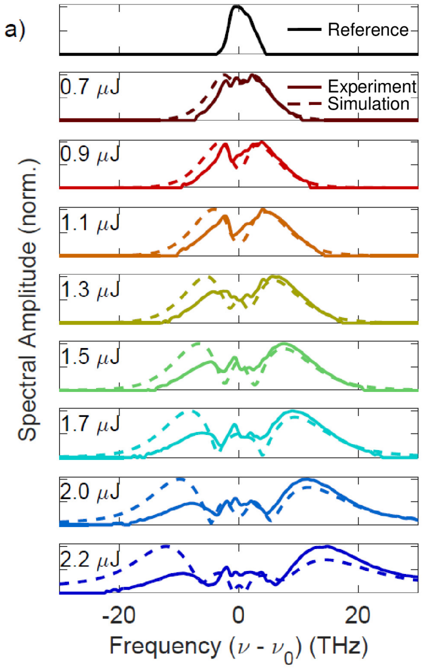
Peak power P_p

- The input power is given by: $P_0(t) = |A(0,t)|^2 = P_p |U(0,t)|^2$
- The nonlinear phase shift is : $\varphi_{NL} = \gamma P_p |U(0,t)|^2 L_{eff}$
- The maximum phase shift is: $\boxed{\varphi_{NL,\max} = \gamma P_p L_{eff}}$

As a result, the spectrum of a wave is broadened with propagation distance







Instantaneous frequency and chirp

The ***instantaneous frequency*** is given by:

$$\omega = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[-\omega_0 + \frac{2\pi}{\lambda_0}(n_0 + n_2 I)L_{eff} + \phi_0\right]$$

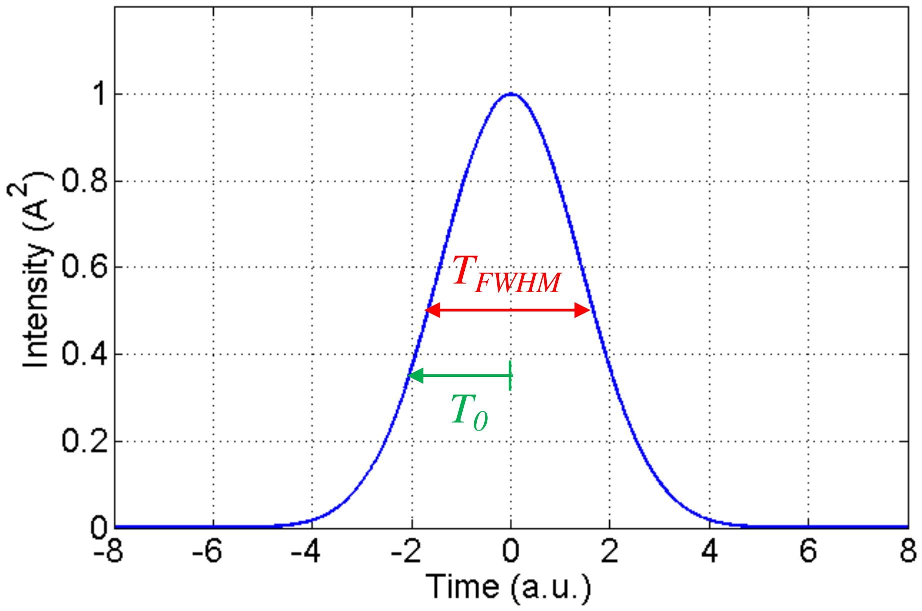
$$\omega = \omega_0 - \frac{2\pi}{\lambda_0} n_2 L_{eff} \frac{dI}{dt}$$

The difference from central value is : $\delta\omega = -\gamma L_{eff}\frac{dP_0}{dt} = -\frac{d\varphi_{NL}}{dt}$

- The time dependence of $\delta\omega$(t) is referred to as ***frequency chirp***

Induced nonlinear chirp

Example of an unchirped input Gaussian pulse:



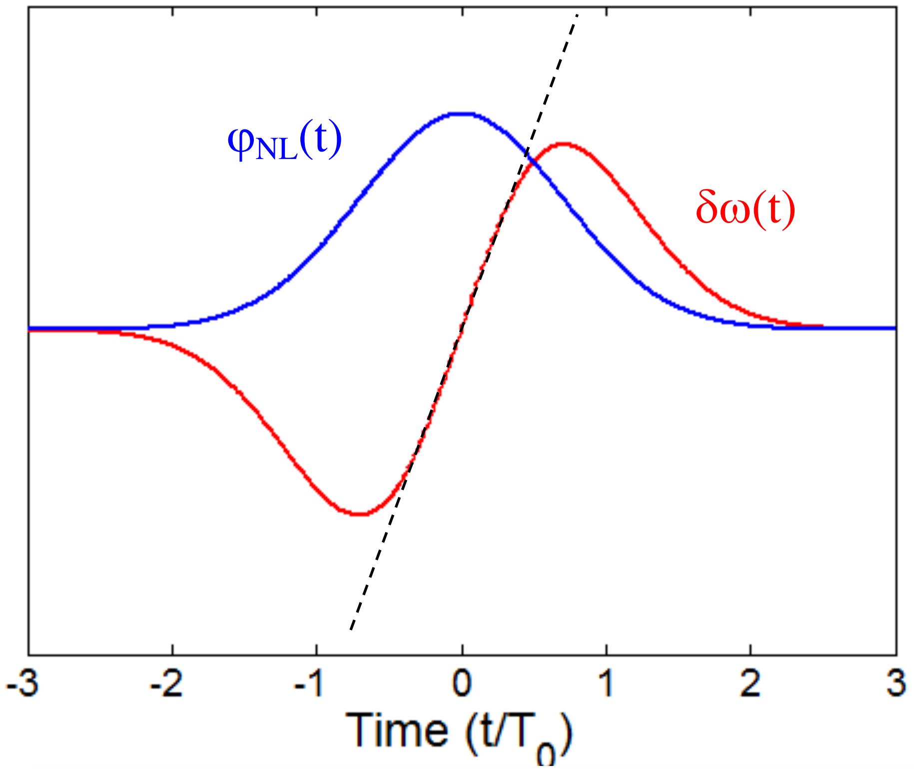
$$A(0,t) = \sqrt{P_p}\exp\left[-\frac{1}{2}\left(\frac{t}{T_0}\right)^2\right]$$

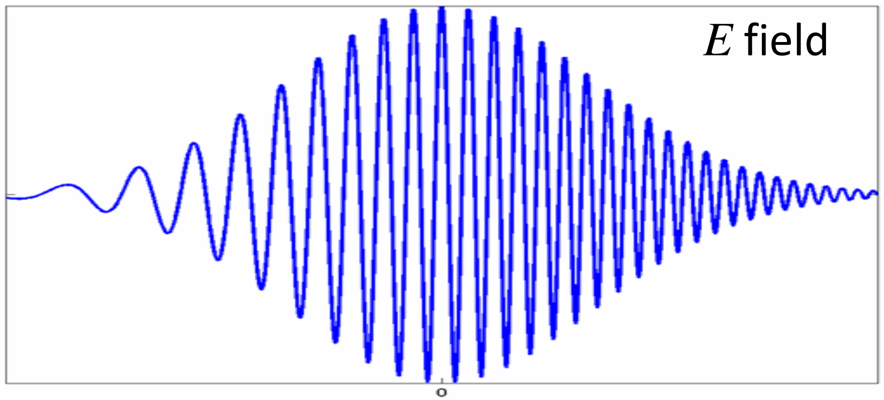
$$P_0(t) = |A(0,t)|^2 = P_p\exp\left[-\left(\frac{t}{T_0}\right)^2\right]$$

$$\varphi_{NL} = \gamma P_p\exp\left[-\left(\frac{t}{T_0}\right)^2\right]L_{eff}$$ *Nonlinear phase shift*

$$\delta\omega = 2\gamma P_p\frac{t}{T_0^2}\exp\left[-\left(\frac{t}{T_0}\right)^2\right]L_{eff}$$ *Induced nonlinear chirp*

SPM impact on pulse





Nonlinear phase shift φ_{NL} is directly proportional to $|A(0,t)|^2$

- Temporal variation identical to pulse shape

Induced chirp is negative near the leading edge and becomes positive

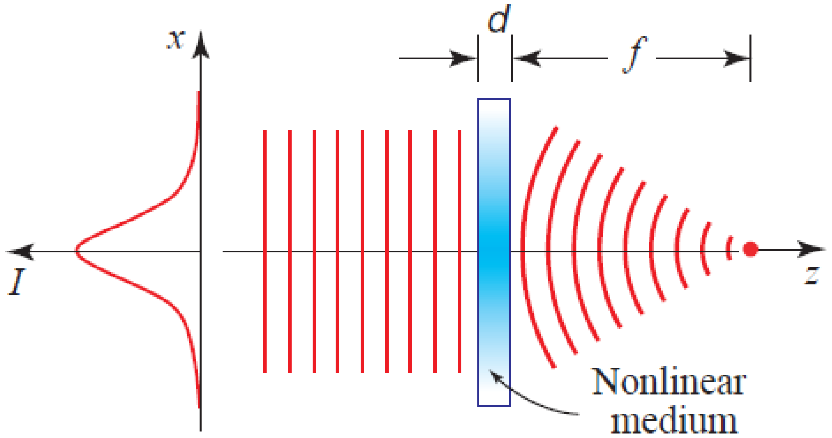
- When intensity *grows*, the frequency shifts to the red,
- When intensity *decreases*, the frequency shifts to the blue.
- Linear up chirp over a large central region

Beam self focussing

Another interesting effect associated with SPM is self-focusing. Consider the beam of light in space transmitted through a nonlinear medium:

- Refractive index change mimics intensity pattern in the transverse plane

$$n(x) = n_0 + n_2^I I$$

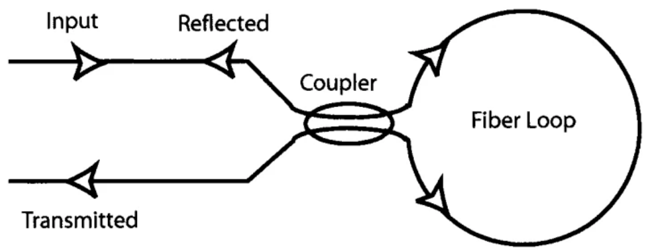


- The nonuniform phase shift causes the wavefront to curve
- Under certain conditions the medium can act as a lens with a power-dependent focal length

Use of SPM – fiber loop mirror

A simple way to realize a reflecting device (i.e. mirror) in fiber optics is to make a *fiber loop.*

- Use a 2x2 directional coupler where the ports on one side are connected to each other



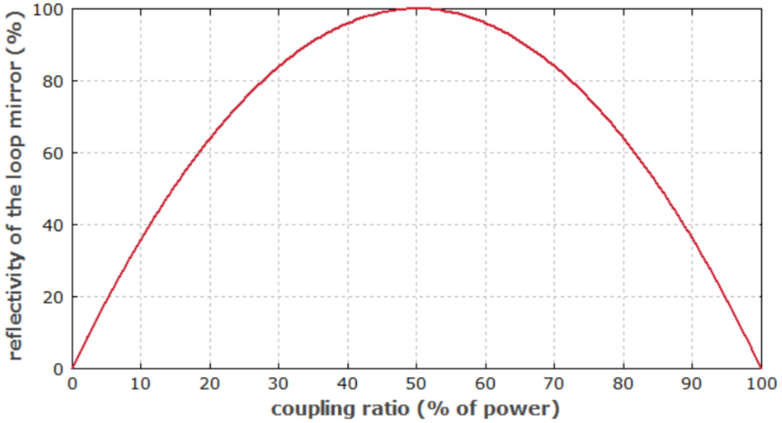
Schematic of an all-fiber Sagnac interferometer acting as a nonlinear optical loop mirror whose transmission depends on launched input power.

Let's first consider a *linear* fiber loop. We assume:

- Single mode fiber, no propagation losses and a maintained polarization state during propagation
- Low optical power so that nonlinear effects are negligible

If the coupler is a 50:50 (3dB coupler):

- Interference conditions lead to all the injected power to go back to the input port: perfect reflector

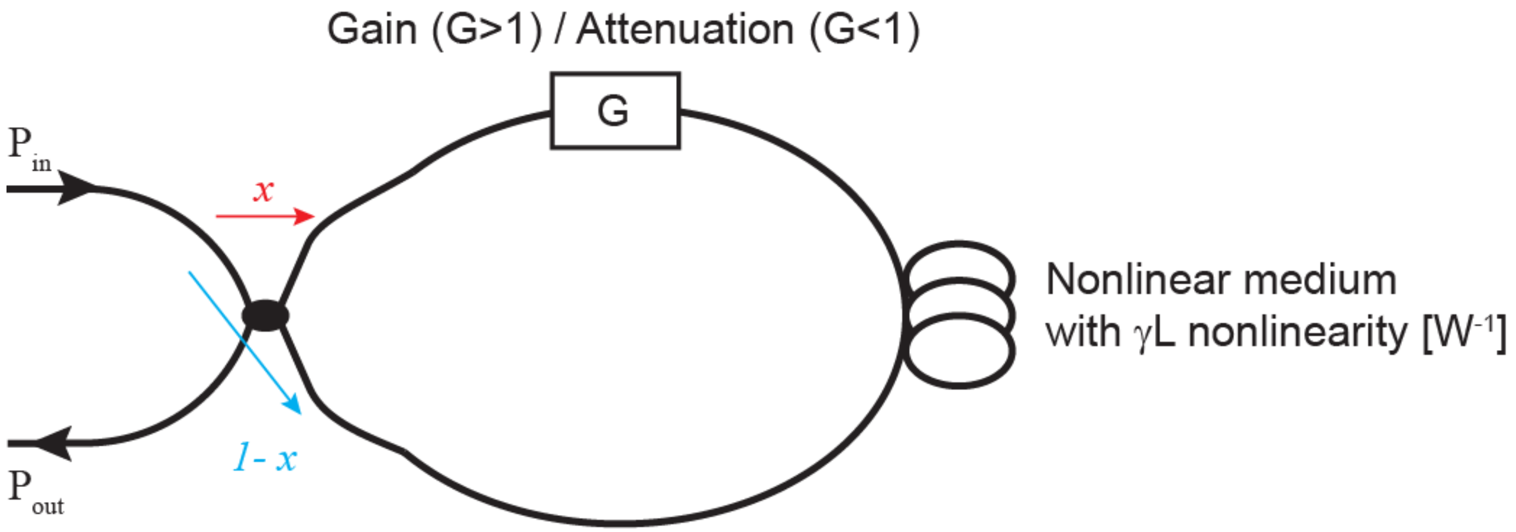


If the power splitting ratio deviates from 50:50, one obtains less than 100% reflectance.

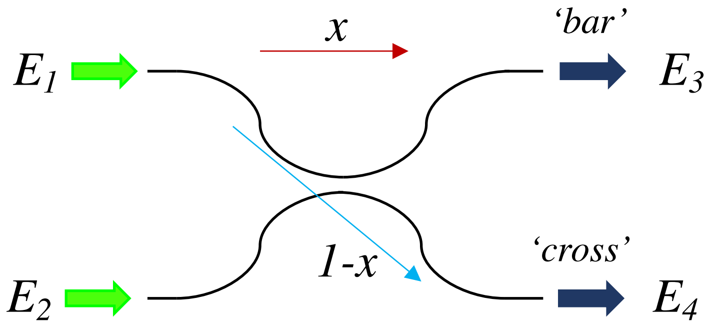
Nonlinear fiber loop mirror (NOLM)

Nonlinearly the effect of propagation will no longer be identical for the two paths if the power ratio of the 2 paths is not 50%

- The phase velocity is intensity dependent, therefore the CW and CCW will experience different phase shift: the device is phase sensitive without the need for an interferometer
- The NOLM is often used for it's thresholding capabilities

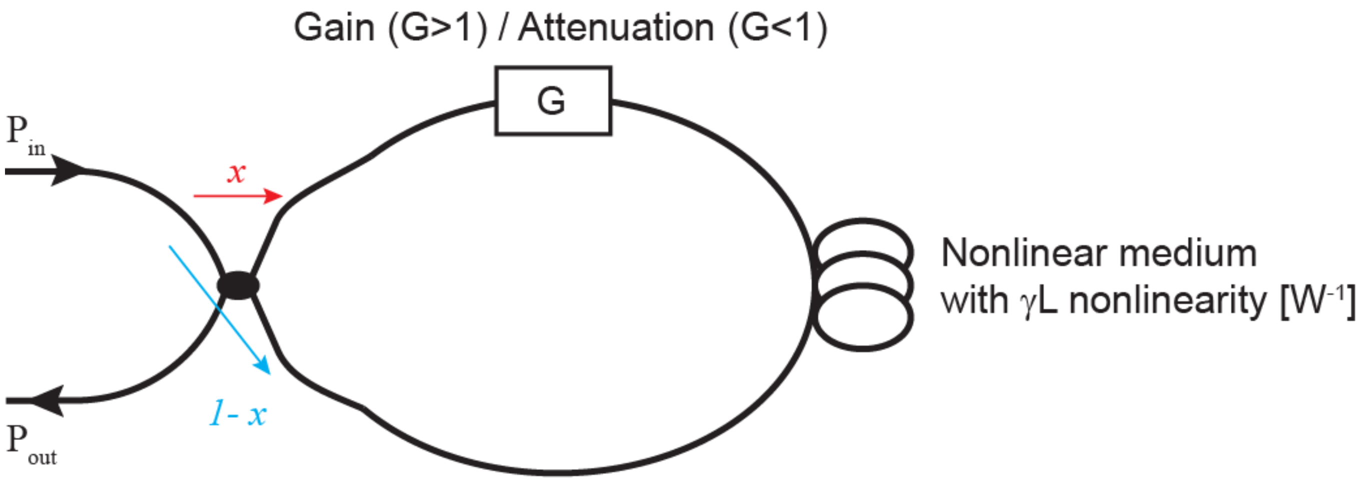


x: power ratio of the coupler

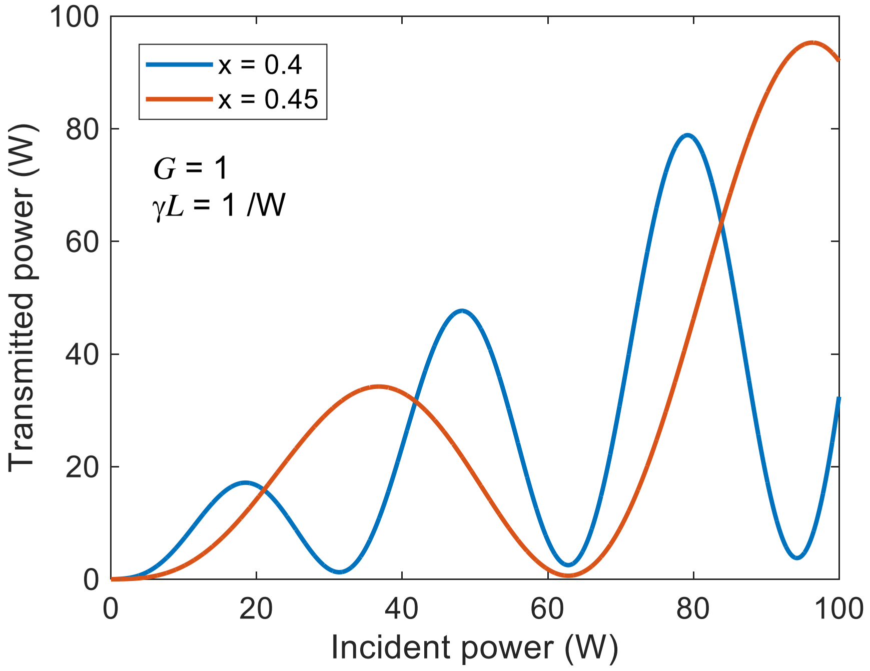


$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \sqrt{x} & i\sqrt{1-x} \\ i\sqrt{1-x} & \sqrt{x} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

Nonlinear fiber loop mirror (NOLM)



x: power ratio of the coupler



$$E_{out,CW} = E_{in} x \sqrt{G} \exp[j\gamma(xGP_{in})L]$$

$$E_{out,CCW} = E_{in}(1-x)\sqrt{G} \exp[j\gamma(1-x)P_{in}L + j\pi]$$

$$P_{out,tot} = P_{in}G[1 - 2x(1-x)(1+\cos\{\gamma P_{in}L(1-x-xG)\})]$$

For x = 0.5, the transmittance is: $T = \frac{P_{out,tot}}{P_{in}} = \frac{G}{2}\left[1 - \cos\left\{\varphi_{NL}\left(\frac{1-\ G}{2}\right)\right\}\right]$

Nonlinearities in optical fibers/waveguides

The lowest order of nonlinearity in silica glass (and most CMOS compatible materials) is of the third order due to non-resonant electronic process

- This leads to a relatively small nonlinear refractive index n_2

However waveguides have several advantages for nonlinear optics

- Strong beam confinement into a small effective area
- Long interaction length and possible small attenuation

This leads to much more nonlinear phase-shifts than using a bulk medium

$\Rightarrow$ great advantage of waveguided nonlinear optics as the required power can be decreased.

Quiz

Silica has n_2 = 2.8 x 10^{-20} m^2/W at 1550 nm

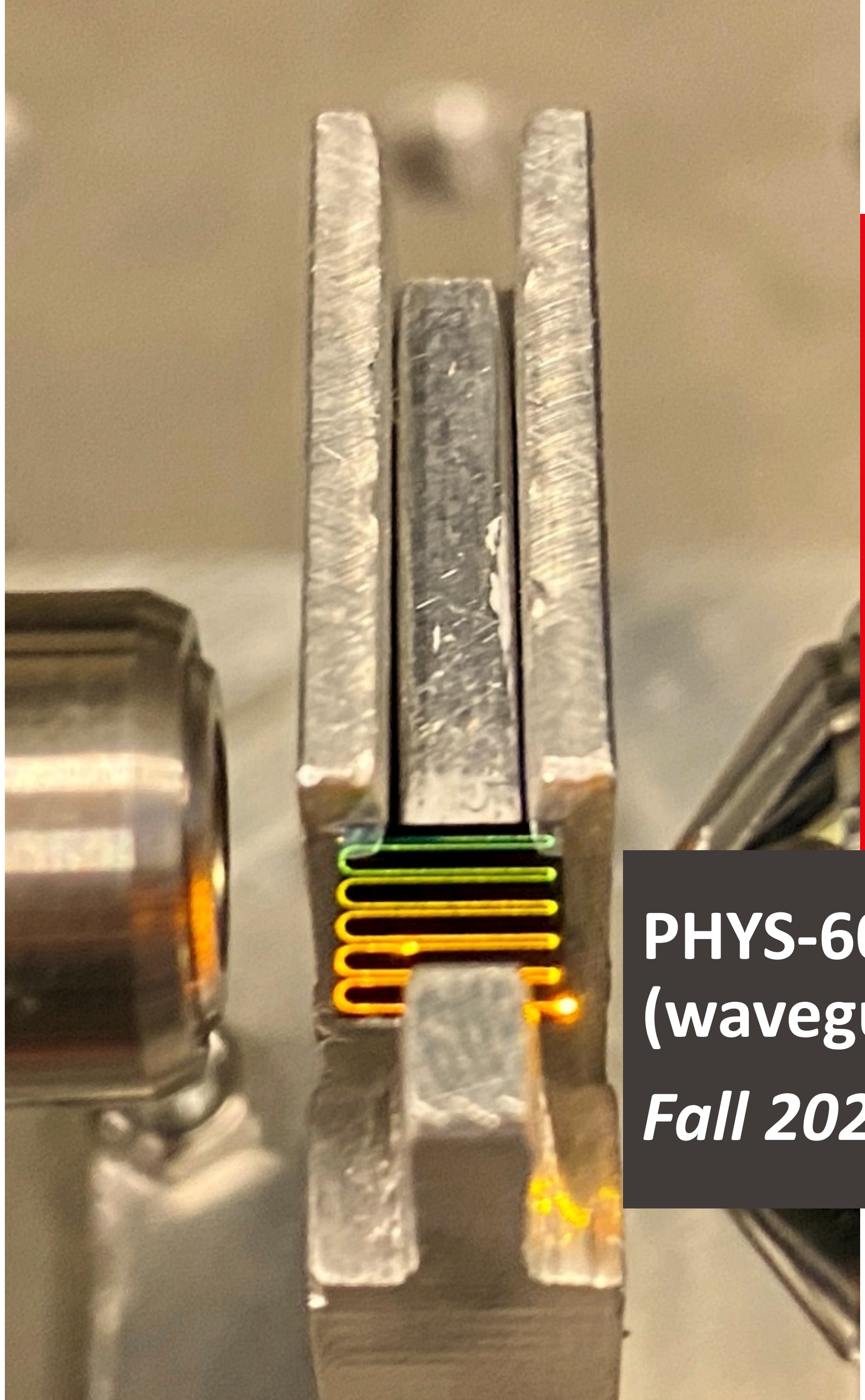
What is the maximum nonlinear phase-shift experienced by a transform-limited Gaussian pulse of peak power 1 mW after

a) Propagation in bulk where the focal point has a surface area of 1 μm^2 and effective propagation around the focal point of 0.5 cm (quite ambitious)

b) In 100 km of optical fibre perfectly loss compensated with an effective area of 50 μm^2.

c) In 100 km of optical fibre with 0.2 dB/km of loss with an effective area of 50 μm^2.

EPFL



Optical Kerr effect

Module 3 – Nonlinear pulse propagation

PHYS-607 : Nonlinear fibre (waveguide) optics

Fall 2022

Ecole Polytechnique Fédérale de Lausanne

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Nonlinear pulse propagation

Reminder

We have seen a (simplified) equation that enables us to solve the mode propagation inside a waveguide:

$$\frac{\partial A}{\partial z}+\beta_1\frac{\partial A}{\partial t}+j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}-\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}=j\gamma|A|^2A-\frac{\alpha}{2}A$$

Changing the static time frame of reference for one that moves at the group velocity ($v_g = 1/\beta_1(\omega)$)

$$\frac{\partial A}{\partial z}+j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}-\frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3}=j\gamma|A|^2A-\frac{\alpha}{2}A$$

Nonlinear Schrödinger equation (NLSE)

Here $T=t-\frac{z}{v_g}$

In silica this equation is valied when T_{FWHM} > 10ps.

Propagation regimes

There are different propagation regimes: depending on the initial pulse width T_0 and on the input peak power P_0 either dispersive or nonlinear effects may dominate upon propagation

Let's consider 2 length scales:

$$L_D = \frac{T_o^2}{|\beta_2|}$$

Dispersion length

$$L_{NL} = \frac{1}{\gamma P_0}$$

Nonlinear length

- For $L \ll L_{NL}$ and $L \ll L_D$: fiber plays a passive role, no dispersive/nonlinear effects upon propagation
- For $L \ll L_{NL}$ but $L \sim L_D$: pulse evolution is governed by group velocity dispersion (GVD)
- For $L \ll L_D$ and $L \sim L_{NL}$: dispersion is negligible compared to nonlinear term (as long as pulse has a smooth spectrum) and pulse evolution is governed by SPM
- For L longer or comparable L_D and L_{NL}: interplay between GVD and SPM.

Dispersion induced pulse broadening

Let's first consider the sole contribution of group velocity dispersion with β_2

- That is considering the waveguides as a *linear optical medium*

We define the unit amplitude envelope U: $A(z,t) = \sqrt{P_0}\exp\left(-\frac{1}{2}\alpha z\right)U(z,t)$

$$\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} = -\frac{\alpha}{2}A \implies \frac{\partial U}{\partial z} = -j\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2}$$

Dispersion is a frequency domain operation - we use Fourier transform method: $\frac{\partial}{\partial z} \Leftrightarrow -j\omega$

$$\frac{\partial \tilde{U}}{\partial z} = j\omega^2\frac{\beta_2}{2}\tilde{U}$$

Solution is given by:

$$\tilde{U}(z,\omega) = \tilde{U}(0,\omega)\exp\left(j\omega^2\frac{\beta_2}{2}z\right) \quad \text{with} \quad \tilde{U}(0,\omega) = \int_{-\infty}^{\infty} U(0,T)\exp(j\omega T)dT$$

Example: Gaussian pulse of unit amplitude

Time: $U(0,T) = \exp\left(-\dfrac{T^2}{2T_0^2}\right)$

Initial spectrum: $\tilde{U}(0,\omega) = \sqrt{2\pi T_0^2}\exp\left(-\dfrac{T_0^2\omega^2}{2}\right)$ with $\Delta\omega_0 = \dfrac{1}{\mathrm{T}_0}$

(1/e bandwidth)

Use:

$$\int_{-\infty}^{\infty} \exp(-ax^2 - 2bx)dx = \sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{a}\right)$$

After propagation: $U(z,T) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \tilde{U}(0,\omega)\exp\left(j\omega^2\frac{\beta_2}{2}z - j\omega T\right)d\omega$

Final profile:

$$U(z,T) = \frac{T_0}{\sqrt{T_0^2 - j\beta_2 z}}\exp\left[-\frac{T^2}{2(T_0^2 - j\beta_2 z)}\right]$$

Try to get this answer yourself!

Pulse broadening

The pulse envelope is evolving following the real and imaginary part of the solution:

$$U(z,T) = \underbrace{\frac{T_0}{\sqrt[4]{(T_0^2)^2 + (\beta_2 z)^2}}}_{\text{Peak amplitude}} \underbrace{\exp\left[-\frac{T_0^2 T^2}{2[(T_0^2)^2 + (\beta_2 z)^2]}\right]}_{\text{Gaussian profile}} \underbrace{\exp\left[-j\frac{\beta_2 z T^2}{2[(T_0^2)^2 + (\beta_2 z)^2]}\right]}_{\text{Frequency chirp}} \underbrace{\exp\left[\frac{j}{2}\tan^{-1}\left(\frac{\beta_2 z}{T_0^2}\right)\right]}_{\text{Phase correction}}$$

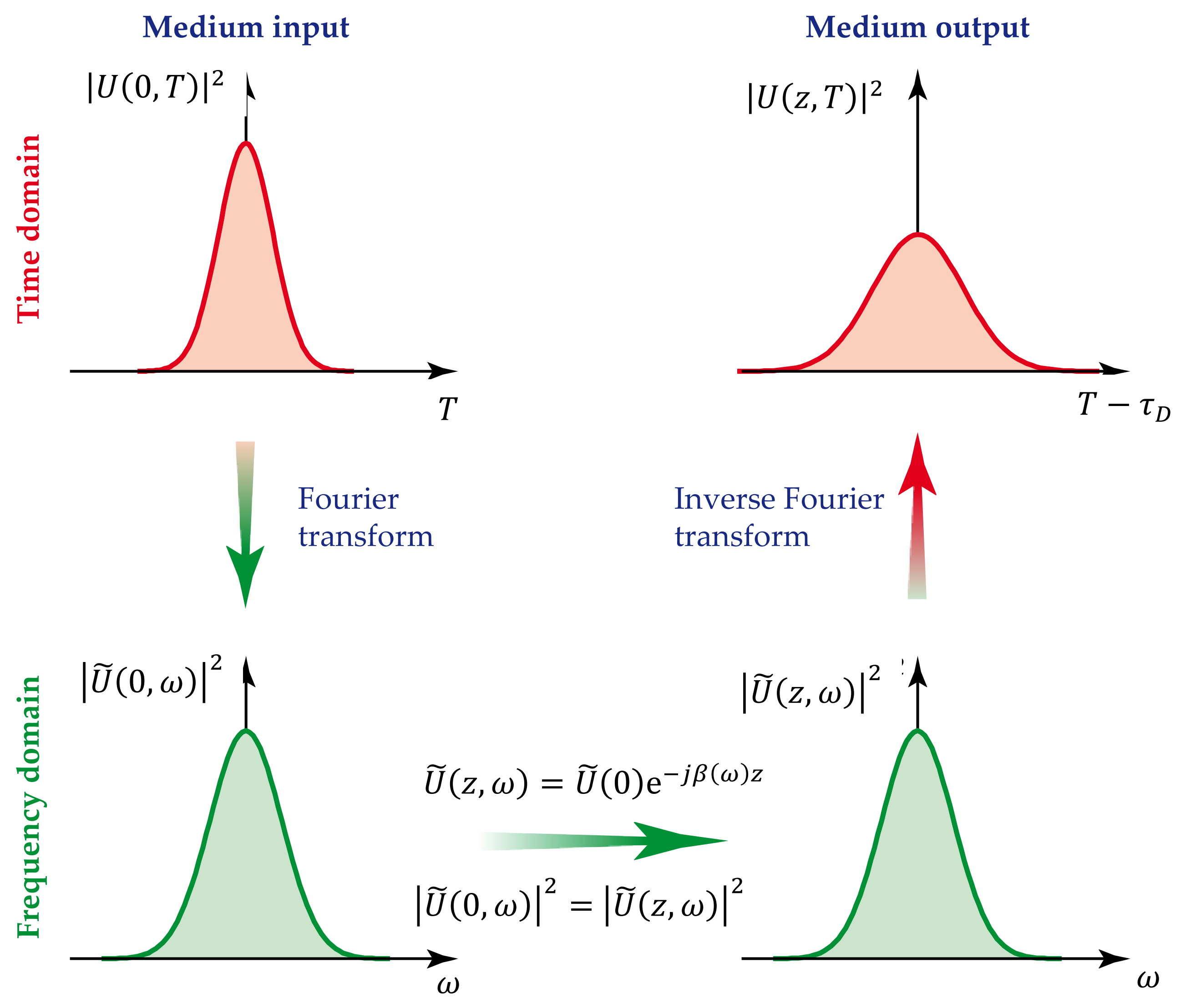
Pulse broadening: $\frac{T_1}{T_0} = \sqrt{1 + \left(\frac{\beta_2 z}{T_0^2}\right)^2} \equiv \sqrt{1 + \left(\frac{z}{L_D}\right)^2}$

Phase shift: $\phi(z,T) = \frac{1}{2}\left[-\frac{\beta_2 z T^2}{[(T_0^2)^2 + (\beta_2 z)^2]} + \tan^{-1}\left(\frac{\beta_2 z}{T_0^2}\right)\right]$

Chirp: $\delta\omega(z,T) = -\frac{d\phi(z,T)}{dT} = \frac{\beta_2 z}{[(T_0^2)^2 + (\beta_2 z)^2]} T$

Try to plot the pulse profile, phase and chirp after various amount of propagation.

Effect of dispersion on Gaussian pulses



- The phase of the different spectral components is changed, however
  - Intensity spectrum remains unchanged.
  - The dispersion is a linear effect, thus does not modify the spectrum.
- The reordering of phases causes a temporal broadening of the pulse.
- This reordering also causes a linear chirp of the instantaneous frequency through the pulse.

Chirped Gaussian pulse propagation

For an unchirped Gaussian pulse, the dispersion induced broadening does not depend on the sign of the GVD parameter β_2. This is not the case if the pulse is initially chirped ...

$$U(0,T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \Longrightarrow U(0,T) = \exp\left(-\frac{(1+jC)}{2}\frac{T^2}{T_0^2}\right)$$

One can show that: $\tilde{U}(0,\omega) = \sqrt{\frac{2\pi T_0^2}{1+jC}}\exp\left(-\frac{T_0^2\omega^2}{2(1+jC)}\right)$ With $\Delta\omega_0 = \frac{\sqrt{1+C^2}}{\mathrm{T}_0}$

(1/e bandwidth)

Also that: $U(z,T) = \frac{1}{\sqrt{Q(z)}}\exp\left[-\frac{(1+jC)T^2}{2T_0^2 Q(z)}\right]$ With $Q(z) = 1 + (C - j)\frac{\beta_2 z}{T_0^2}$

And finally that : $\frac{T_1}{T_0} = \sqrt{\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}$ $C_1 = C + (1+C^2)\frac{\beta_2 z}{T_0^2}$ ***Try to show this!***

Summary of Gaussian pulse propagation

Unchirped pulse (C = 0): broadens monotonically by the same amount in normal & anomalous dispersion regimes

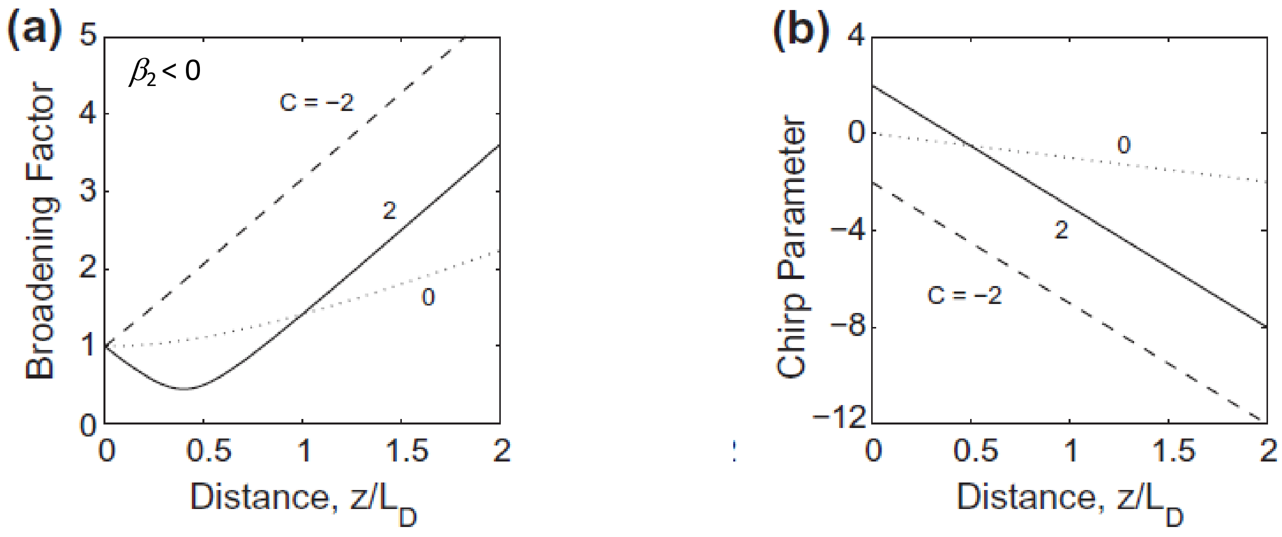
Chirped pulses behave differently depending on the sign of $\beta_2 C$:

- $\beta_2 C > 0$: broadens monotonically faster than an unchirped pulse.
- $\beta_2 C < 0$: initially compresses up to z_{min}, reaching a pulse width of T_1^{min}, then broadens

$$z_{min} = \frac{|C|}{(1+C^2)} L_D$$

$$T_1^{min} = \frac{T_0}{\sqrt{1+C^2}} = \frac{1}{\Delta\omega_0}$$

You can show this as well



Impact of higher order dispersion (β_3)

Still using the unit-amplitude envelope with attenuation previously defined, we have:

$$\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} = -\frac{\alpha}{2}A \Longrightarrow \frac{\partial U}{\partial z} = -j\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 U}{\partial T^3}$$

Following the same Fourier-transform technique, we get:

$$\widetilde{U}(z,\omega) = \widetilde{U}(0,\omega)\exp\left(j\omega^2\frac{\beta_2}{2}z + j\omega^3\frac{\beta_3}{6}z\right)$$

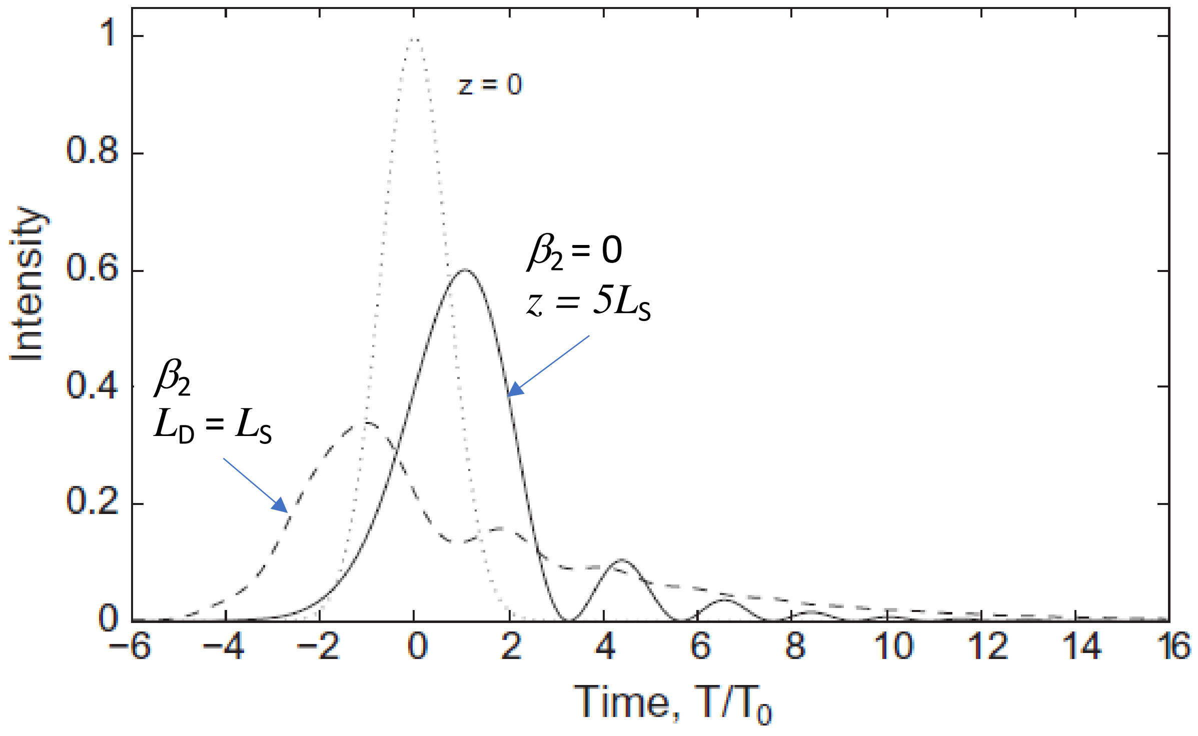
$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widetilde{U}(0,\omega)\exp\left(j\omega^2\frac{\beta_2}{2}z + j\omega^3\frac{\beta_3}{6}z - j\omega T\right)d\omega$$

Example of the Gaussian pulse

We can input once again $\tilde{U}(0, \omega)$ of a Gaussian pulse and apply the IFT to get the pulse in time.

Pulse evolution will depend on the relative importance of β_2 and β_3. We thus define a dispersion length associated with the TOD:

$$L_S = \frac{T_o^3}{|\beta_3|}$$



Pure nonlinear propagation

$$\frac{\partial A}{\partial z} = j\gamma|A|^2 A \implies A(z,T) = A(0,T)\exp(j\gamma|A|^2 z)$$

Reminder

As seen before:

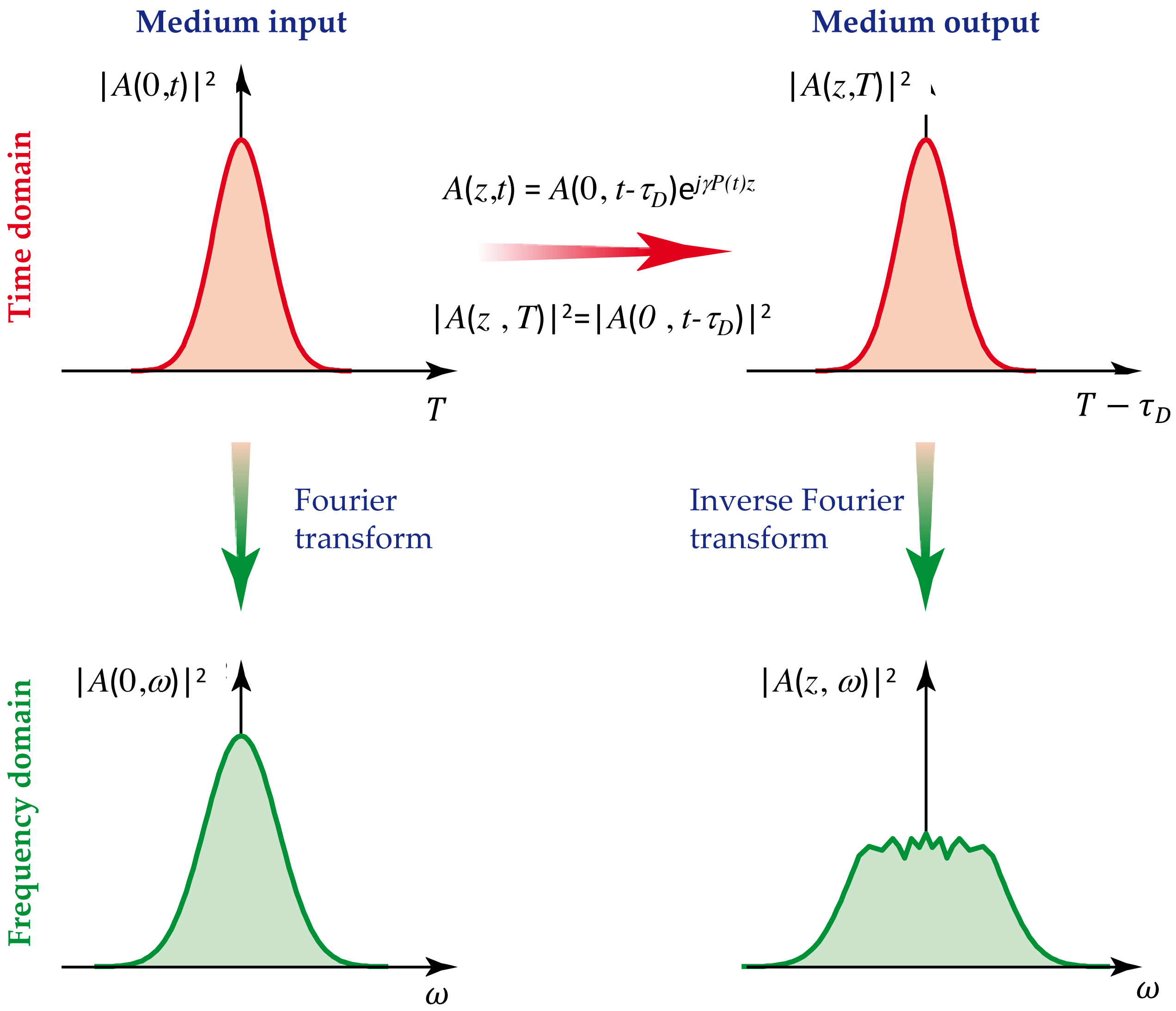
- Propagation in a loss-free nonlinear medium provides a phase shift that accumulates proportional to power and distance. Becomes significant after the nonlinear length
- The nonlinear phase shift induces a chirp:

$$\delta\omega = -\frac{\partial \phi_{NL}}{\partial T} = -\gamma z \frac{d|A|^2}{dT}$$

- The spectrum of a wave is broadened with propagation distance
- The maximum phase shift is

$$\phi_{NL,max} = \gamma P_0 L_{eff}$$

Kerr effect on Gaussian pulses



- The self phase modulation has no impact on the intensity temporal distribution, as any phase modulation. The pulse shape remains unchanged.
- However, this modulation causes a pulse spectral broadening, that is typical of a nonlinear effect.
- This modulation also induces a chirp of the instantaneous frequency through the pulse.

Summary of lengths

$$L_D = \frac{T_o^2}{|\beta_2|}$$ Distance for which $T_1 = \sqrt{2} T_0$

$$L_S = \frac{T_o^3}{|\beta_3|}$$ Distance for which the effect of β_3 is significant

$$L_{NL} = \frac{1}{\gamma P_p}$$ Distance for which $\phi_{\mathrm{NL,max}}$ = 1 rad

$$L_{eff} = \frac{1 - \exp(-\alpha L)}{\alpha}$$ Distance for equivalent nonlinear phase-shift but in a lossless medium

Summary of ‘single’ regimes

	Dispersion only	Nonlinearity only
Phase shift ($e^{j\phi}$)	$\phi_{disp}(L,\omega)=\begin{bmatrix} \frac{1}{2}\beta_2(\omega-\omega_0)^2 \\ +\frac{1}{6}\beta_3(\omega-\omega_0)^3 \\ +\ldots \end{bmatrix} L$	$\phi_{NL}(L,T)=\gamma P L_{eff}$
Power vs. time	Temporal broadening and stretching	No changes to the power profile
PSD vs. frequency	No changes to the power spectrum	Spectral broadening and stretching

Propagation in nonlinear and dispersive media

Reminder

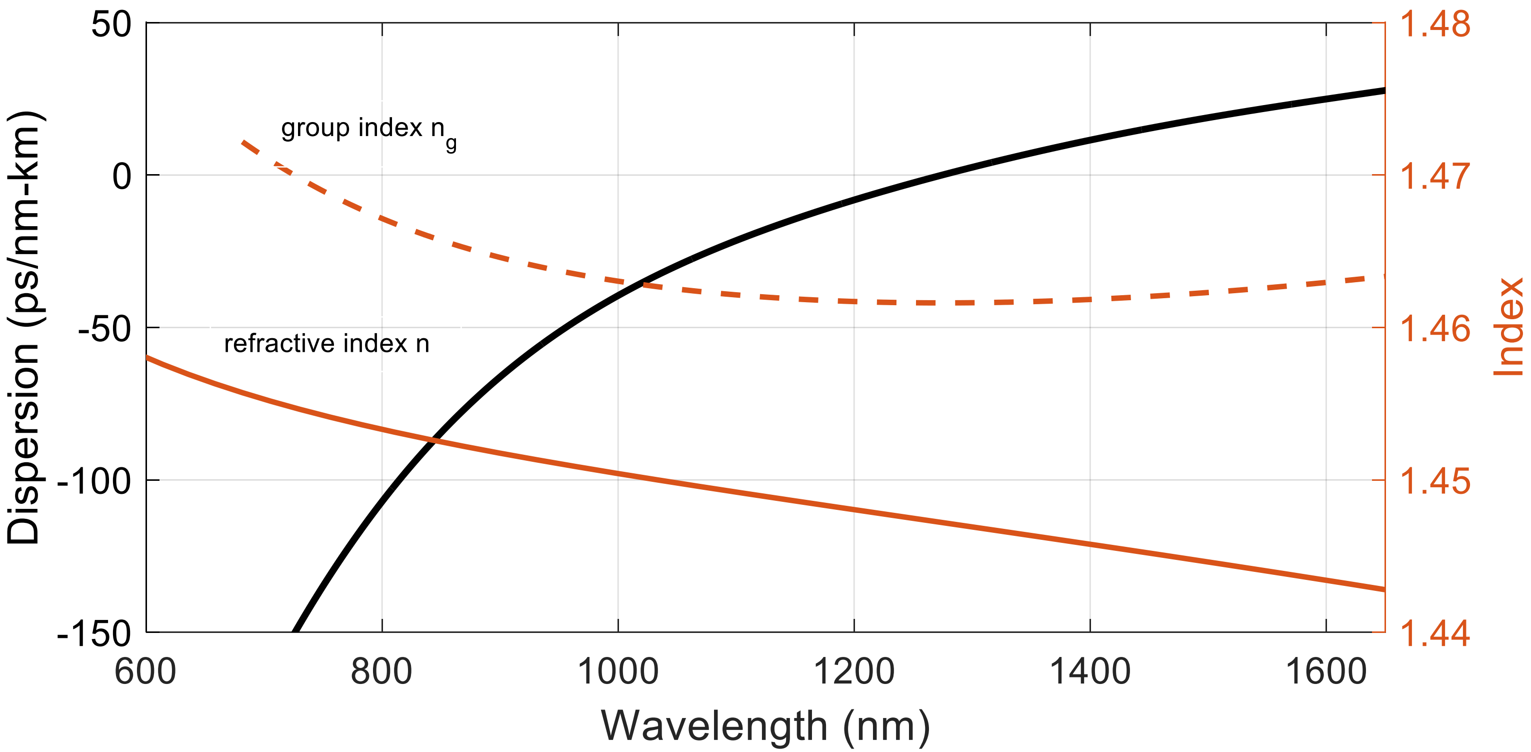
The group velocity dispersion β_2 can be either positive (normal dispersion) or negative (anomalous dispersion)

$$n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega_j^2 - \omega}$$

$$n_g(\lambda) = n - \lambda \frac{dn}{d\lambda}$$

$$D = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

$$\beta_2 = -\frac{\lambda^2}{2\pi c} D$$



The sign of the GVD is important when combined with nonlinear propagation

The N number (the soliton number)

The dimensionless N number is defined as:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_p T_0^2}{|\beta_2|}$$

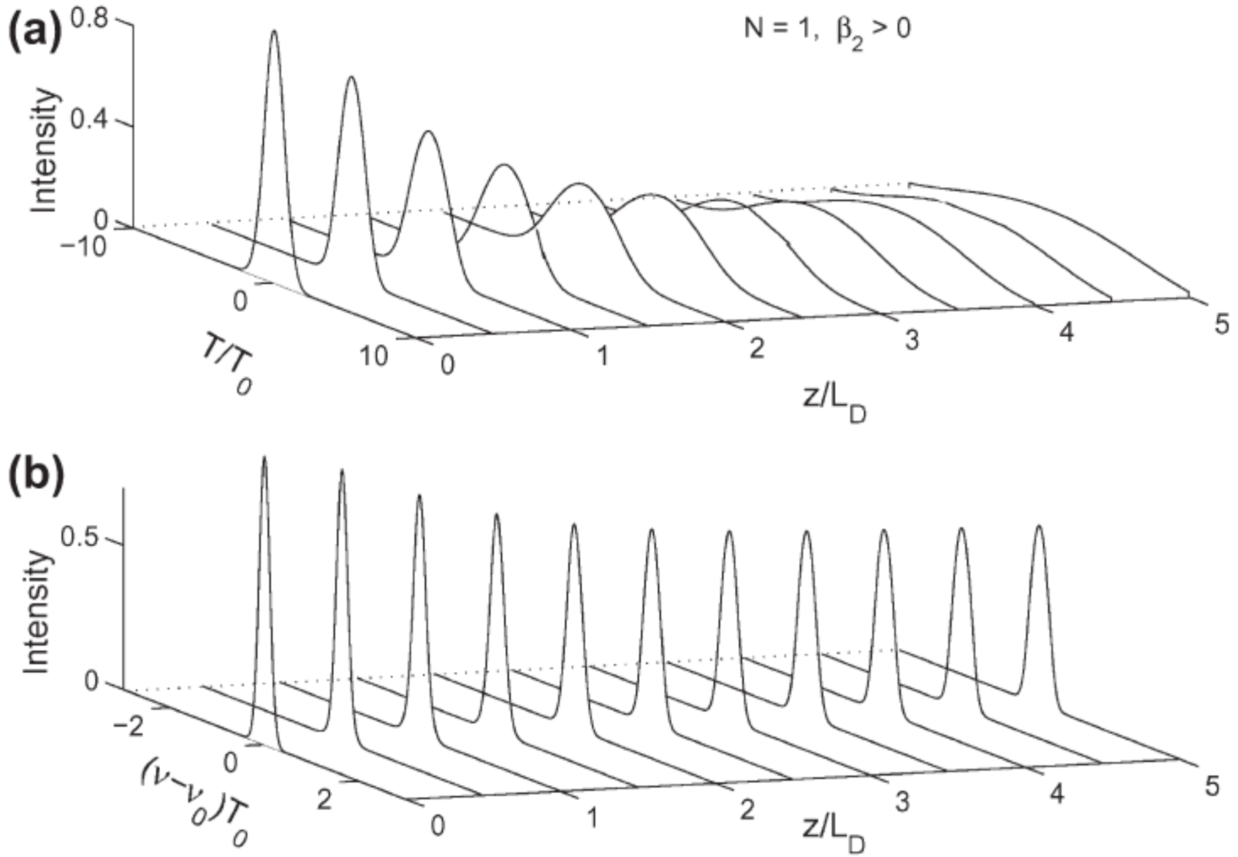
This parameter governs the relative importance of dispersion (GVD) and nonlinearity (SPM) on a pulse as it propagates

- For $N \ll 1$: dispersion dominates
- For $N \gg 1$: SPM dominates
- For $N \sim 1$: SPM and GVD play an equally important role

Propagation in normal dispersion ($\beta_2 > 0$) for $N = 1$

Normal dispersion and nonlinearity both chirp the center of the pulse in the same way

- Positive chirp leading to a red shift

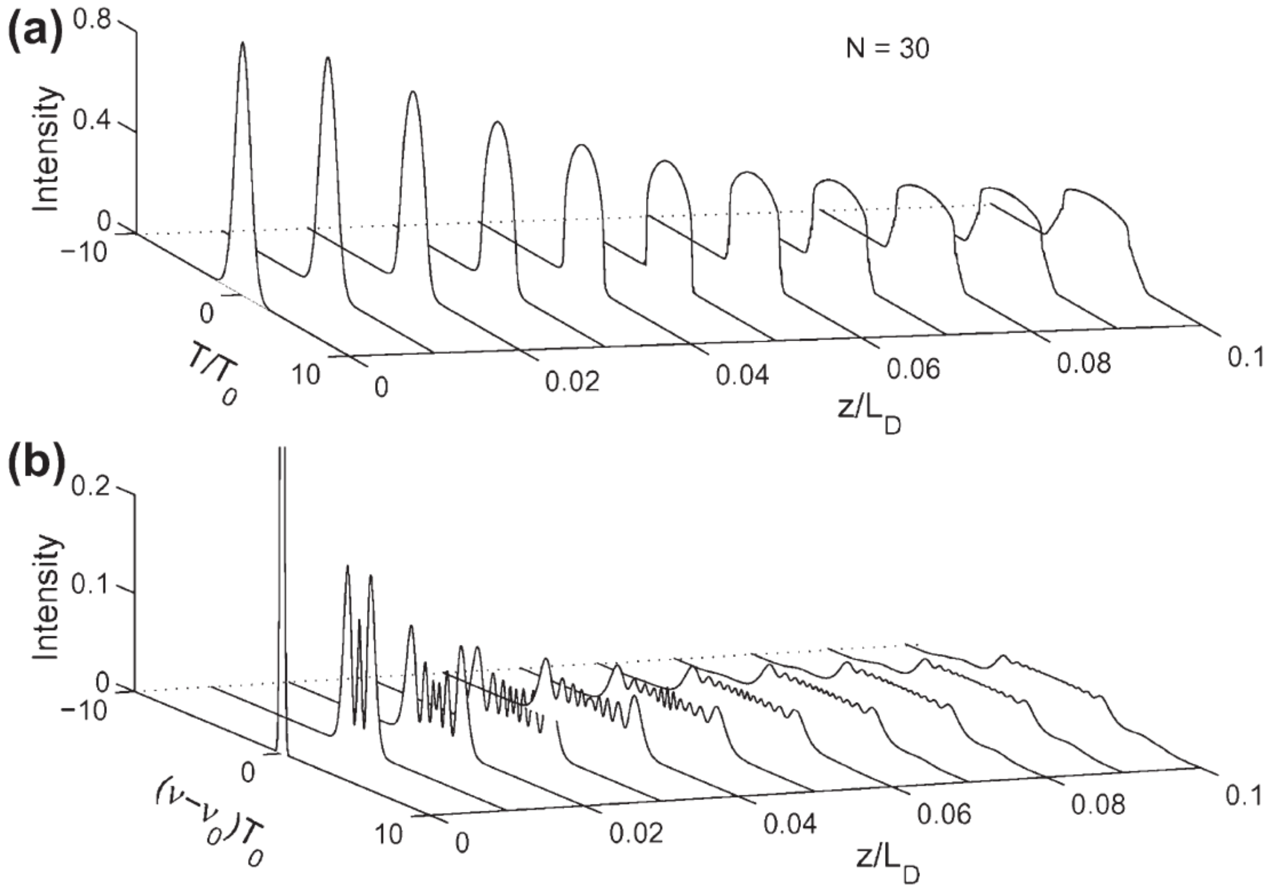


From Agrawal

Propagation in normal dispersion ($\beta_2 > 0$) for $N >> 1$

The strong nonlinearity in normal dispersion can lead to **optical wave breaking** phenomenon.

- Rapid oscillations near the pulse edges accompanied by sidelobes in the spectrum
- Central part of the spectrum modified from pure SPM: minima are not as deep as expected.



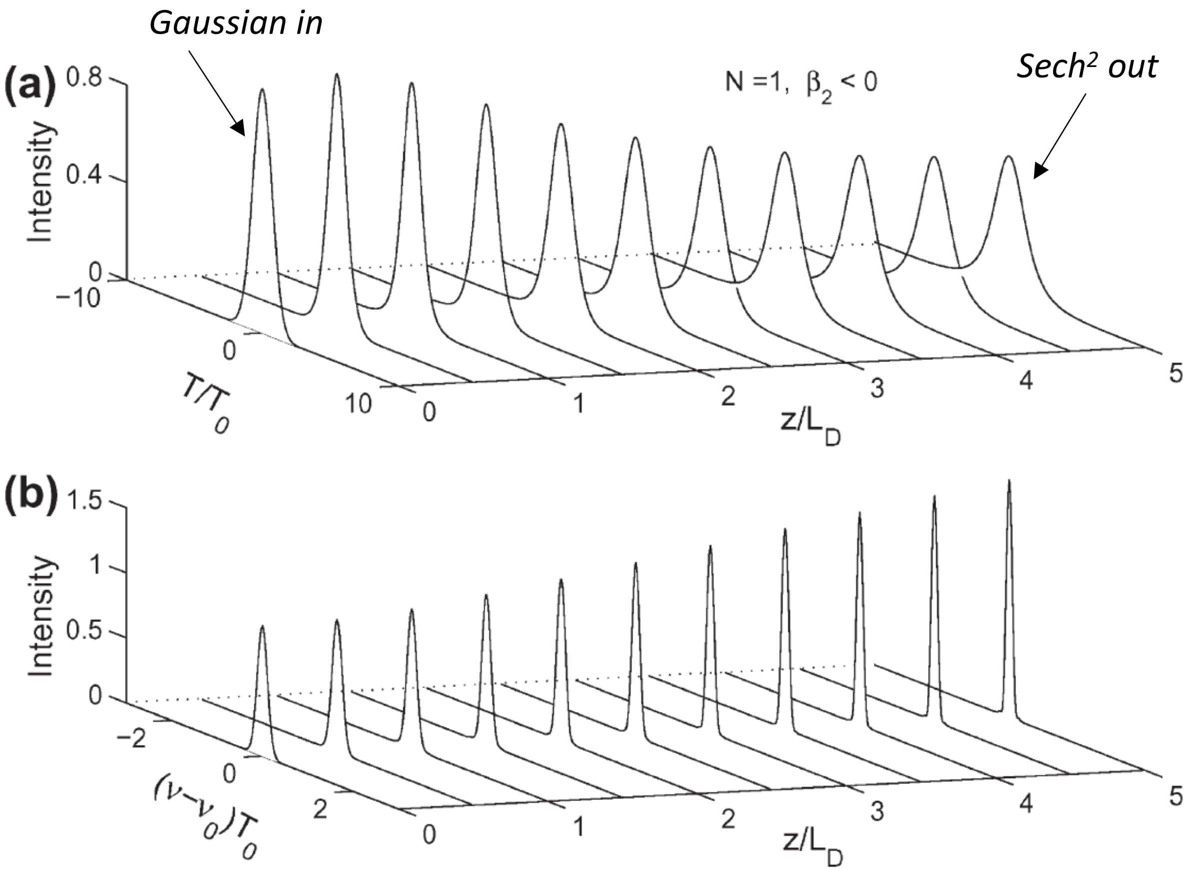
From Agrawal

- **Frequency-shifted light** in the leading and trailing edges of a pulse **overtakes unshifted light** in the pulse tails.
- Mixing of these overlapping frequency components generates sidelobes on the pulse spectrum.

Propagation in anomalous dispersion ($\beta_2 < 0$) for $N = 1$

Anomalous dispersion and nonlinearity both chirp the pulse in opposite manners on every infinitesimal incremental propagating distance.

- In equilibrium it leads to the **soliton**



From Agrawal

Quiz

Nonlinear effects which are defined by the intensity dependent refractive index of the waveguide are called:

a) Raman effect

b) Dispersive effect

c) Kerr effects

d) Soliton effect